**The Ratio and Root Tests**

**Theorem:** (The ratio test)

Let be a series with positive terms and suppose that

.

Then

1. The series converges if .
2. The series diverges if  or is an infinite.
3. The test fails if .

**Example:**

Which of the following series converge, and which diverge

1.  (2)  (3)  (4) 

**Solution:**

(1)



then the series  converges.

(2) 



 

then the series  diverges.

(3) 



 

then the series  converges.

(4) 

 then the series  converges.

**Theorem:** (The root test)

Let be a series with positive terms and suppose that

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Then

1. The series converges if .
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**Example:**

Which of the following series converge, and which diverge

(1)  (2)  (3) 

**Solution:**

(1) 



then the series  converges.

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then the series  diverges.

(3) 



then the series  converges.

**Exercises**

Which of the following series converge, and which diverge

     

  